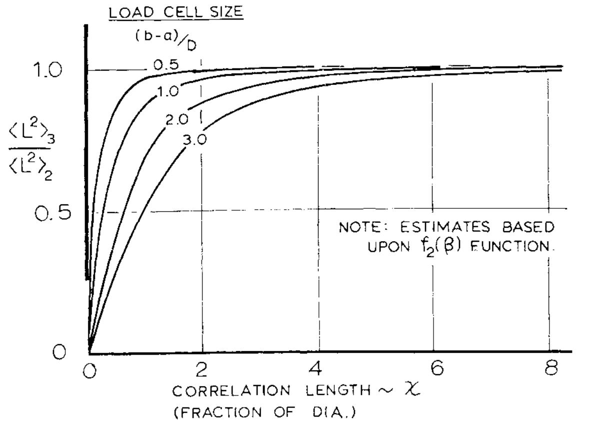


**Fig. 4 Transducer effectiveness (exponential $\psi_3$ function).**

output would be increased by approximately 63% relative to the original reading.

### Concluding Remarks

The spatial resolution achieved with a sensing element depends upon its actual size relative to a cross-correlation length scale. The sample situations considered herein provide one with a quantitative measure of this relationship. In addition, recent improvements in data retrieval and analysis methods make feasible the consideration of greater statistical detail, which in turn requires that more attention be paid to the spatial resolution of the sensing elements.

Recent research on unsteady loads about bluff bodies has led to a much clearer understanding of three-dimensional flow traits and the manner in which motion can act to unify flow features which otherwise would be disorderly. However, the details of how motion acts to increase section loadings and improve spatial correlations still remains to be defined, and in particular, it is not clear whether this will only occur very near a particular nondimensional frequency that depends upon geometrical shape. When these matters are understood, it will be necessary to have workable methods for predicting structural response to random, motion-dependent inputs.

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# Nonlinear Analysis of a Launch Vehicle Attitude Control System


Sherman M. Seltzer*
*George C. Marshall Space Flight Center, Huntsville, Ala.*


An attitude control system for a large space vehicle was postulated for use during ascent through the denser portion of the Earth's atmosphere. Subsequently this system was modified by the addition of a nonlinear element and analyzed to determine its suitability for continued use after the vehicle had risen above the denser portion of the atmosphere and the guidance loop had closed.[1] Using a Nyquist plot and a describing function linearization, it was determined that a range of control system gains could be chosen that precluded limit cycle operation and ensured stable operation. The low pass nature of the transfer function under investigation lends itself well to utilization of describing function techniques. Subsequent analysis has indicated that, due to some incorrect considerations, there was a range of initial conditions that could excite unstable sustained oscillation. In this Note, this range of initial conditions will be portrayed in a parameter space, and it will be shown how this range can be computed numerically. In addition, a new result will be shown in the sense that a stable region of two control system gains will be established using a single unified method for both the nonlinear and the linear portions of the analysis without recourse to the Nyquist plot previously used. The parameter method provides adjustment of two (rather than one, as with the Nyquist technique) gains in the investigation of sustained oscillations and stable operation which is a significant advantage in this problem.

As shown, the transfer function of the linear portion of the system is

$$G(s) = \frac{(a_0s^2 + k_1s + k_2)(s^2 - c)}{s^3(s^3/\omega^2 + 2\zeta_e s^2/\omega_e + s + a_1c)} \tag{1}$$

The characteristic equation of the linearized system (Fig. 1, Ref. 1) is

$$B(s) + G_D(A)C(s) = 0 \tag{2}$$

where $G_D(A)$ is the describing function of the nonlinear element with a saturation characteristic. Specifying the variable parameters as $\epsilon \equiv k_2G_D(A)$ and $\eta \equiv G_D(A)$, one obtains from Eq. (2) the $\zeta = 0$ curve on the parameter plane diagrams of Fig. 1. As known,[2] the existence of limit cycles is indicated at intersections of the $\zeta = 0$ curve which determines the stable region and the $M$-locus which represents the variation of the describing function $G_D(A)$ in the parameter plane. The other bound of the stable region is defined by the real root boundary which in this case is the $\eta$-axis. Stable operation is predicted for those amplitudes (values of $A < A_{LC}$ where $A_{LC}$ is the amplitude of the limit cycle) for which a portion of the describing function line lies within the stable region. If $A$ becomes large enough ($A > A_{LC}$) to cause operation to occur outside the stable region, instability is indicated. Since the slope of the describing function line is $k_2$, it is seen that a greater portion of that line (and hence a wider range of initial conditions, i.e., initial amplitudes, $A$) can be made to lie within the stable region as $k_2$ is decreased in magnitude. Hence a restriction on admissible initial conditions is apparent that was not brought out in Ref. 1.

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* Senior Research Engineer, Astrionics Laboratory. Associate Fellow AIAA.

Since small values of $k_2$ permit more admissible initial conditions, it is appropriate to determine the effect on system characteristics if $k_2$ is set equal to zero.

Redefining the adjustable parameters to be portrayed as $\alpha \equiv a_0 G_D$ and $\gamma \equiv k_1 G_D$, Eq. (2) is solved for $\alpha$ and $\gamma$, and the complex root stability boundary (double cross-hatching) is plotted on the $\alpha$–$\gamma$ parameter plane (Fig. 2). The real root stability boundary (single cross-hatching) also is determined from Eq. (2), yielding the line $\gamma = 0$. The describing function curve again is a straight line, $\gamma = k_1\alpha/a_0$, on the parameter plane. Values for the control gains may be chosen that will keep the entire line within the stable region of the parameter plane, precluding sustained oscillations and indicating stable operation for the conditions portrayed. Further, a curve corresponding to $\zeta = 0.5$ may be determined by setting $s = -\zeta\omega_n + i\omega_n(1 - \zeta^2)^{1/2}$ in Eq. (2) and repeating the procedure outlined earlier to determine $\alpha$ and $\gamma$. Values of $\alpha$ and $\gamma$ may be chosen so that the describing function line terminates on the $\zeta = 0.5$ curve at a desired control frequency (indicated in parentheses). However, the parameter $c$ is not constant throughout flight.[1] During most of the upper stage flight it has a value of approximately 0.7, rising to 2.5 for a short time duration. A stability region is shown on Fig. 2 for that higher value of $c$, and this additional constraint is imposed on the selection of the terminus for the describing function line. If values of $-0.415$ and $-0.110$ are chosen for $\alpha$ and $\gamma$ and the slope $K$ of the saturation limiter is set at unity, limit cycle operation for all initial values of $\dot{\sigma}$ (input to the saturation characteristic) is obviated and stable operation is indicated.

The ratio $A/S$ equals unity at the describing function line terminus; if $S = 1$, then $A$ is limited to 1°/sec (as desired). Good damping characteristics are indicated except for the short time that $c$ approaches values of 2.5. The indicated value of $\omega = 0.433$ for $c = 0.7$ (and $\omega = 7$ for $c = 2.5$) provides an acceptable frequency separation from the guidance and bending frequencies. Hence it is seen that stable operation and a close match to the desired design characteristics established for control frequency, damping ratio, and saturation limit are indicated by this analysis. The simplicity of analyzing both linear and nonlinear operation in the parameter plane is noteworthy, as is the fact that the effect on linear and nonlinear stability of adjusting two control gains (in this case $a_0$ and $k_1$) can be seen directly (whereas the Nyquist plot used in Ref. 1 provides this visibility for only one gain at a time). The effect on vehicle performance of setting $k_2 = 0$ will of course be degrading. However, $k_2 \neq 0$ during ascent through the atmosphere, so the effect of wind on drift is still suppressed. After $k_2$ is set equal to zero, the effects on steady state drift may be estimated analytically through application of the final value theorem. These estimates provide an upper bound for they assume open-loop guidance, whereas in reality, the guidance loop will be closed

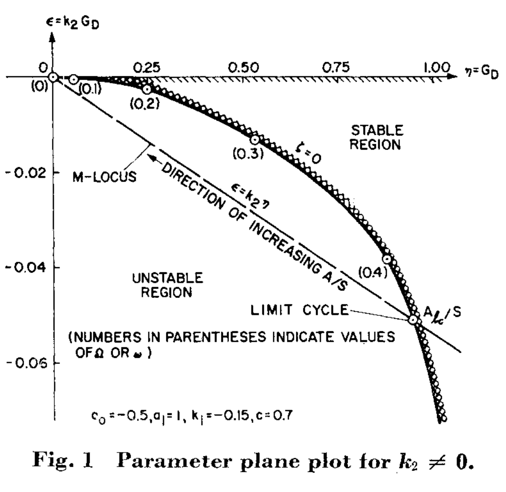


Fig. 1 Parameter plane plot for $k_2 \neq 0$.

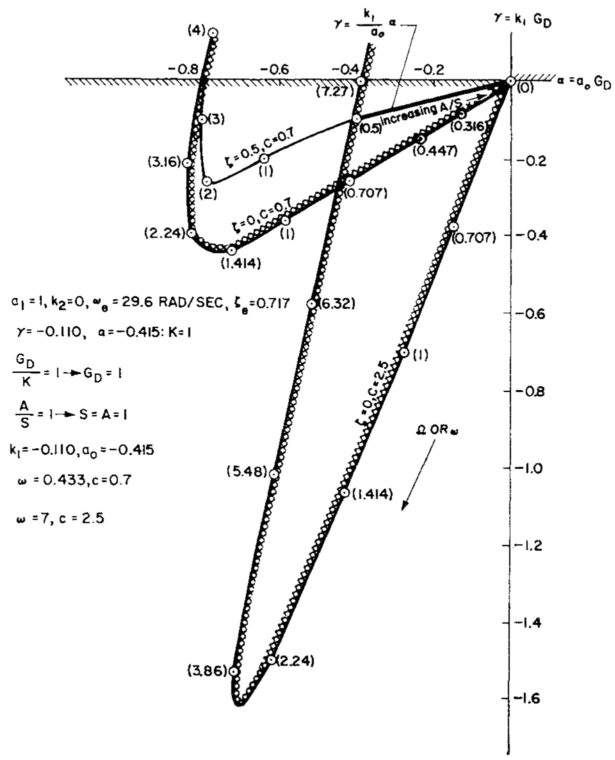


Fig. 2 Parameter plane plot for $k_2 = 0$.

when $k_2$ is set equal to zero. While a finite value of $k_2$ will drive to zero the drift velocity due to wind, center of gravity offset, uncertainties in thrust vector direction due to hardware tolerances, and errors in reading the gimbal angles of the stable platform, setting $k_2 = 0$ will result in finite but small values for the first three of these drift producers and a zero value for the fourth.[3]

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# Iterative Computation of Initial Quaternion

Itzhack Y. Bar-Itzhack*
*Bellcomm Inc., Washington, D.C.*

When quaternions are used to compute the time varying transformation matrix between two Cartesian coordinate systems,[1,2] the initial quaternion has to be known. In most of the cases the initial transformation matrix, rather than the initial quaternion, is known, and the latter has to be computed. In this Note, an iterative technique for computing the quaternion that corresponds to an orthogonal matrix is presented. This technique is based on a known, self-alignment technique of an analytic platform. In its

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* Member of the Technical Staff, Space Vehicle Dynamics Department, Control Group.